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WŁADYSŁAW NATANSON (1864-1937)

For the 110th anniversary of formulation of the thermodynamics of irreversible processes and the 95th anniversary of formulation of quantum statistics

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Abstract

Władysław Natanson, born in Warsaw in 1864, developed his activity as the professor of theoretical physics at Jagellonian University in Cracow. His early papers concerned the kinetic theory of gases and thermodynamics. In 1896 he formulated the principles of thermodynamics of irreversible processes. In 1911 he formulated the quantum statistics by considering the statistics of indistinguishable particles. Both formulations were discoveries which went ahead of contemporary physics. Later Natanson made research in optics taking as the basis the theory of electrons, because he did not adopt the old quantum theory. He approved of quantum mechanics and investigated the wave aspect of matter. He also wrote essays on history and methodology of physics and textbooks of physics.

1 Władysław Natanson's life

Władysław Natanson was a professor of theoretical physics at Jagellonian University in the years 1890-1934 [1.1]-[1.7]. He was born in 1864 as a son of Warsaw physician Ludwik Natanson, who was the author of scientific papers in medicine, psychology and pedagogics. At his father's home young Natanson lived in the scientific atmosphere and admiration of science. From his childhood he showed great abilities for exact sciences. Having graduated from a Warsaw gymnasium, he studied physics in St. Petersburg and then he worked at Cavendish Laboratory in Cambridge. There he had contacts with the great English physicists J.J. Thomson and G. Stokes. He gained his doctorate in Dorpat (Estonia) in 1887. Later he studied in Graz at Ludwig Boltzmann, where he tried, unsuccessfully, to make habilitation. Then he returned to Warsaw, where he wrote a textbook *Introduction to theoretical physics* [1.8], which appeared in 1890.

Since the Russian university in Warsaw was boycotted by Poles, Natanson did not find proper conditions for scientific work there. In 1890 he went to Cracow. Encouraged by Professor Witkowski he made his habilitation in 1891 and the next year he obtained the post of Assistant Professor of mathematical physics. In 1884 he was appointed associated professor and the head of the chair of theoretical physics at Jagellonian University. In 1993 he became a full professor. He was active at Jagellonian University until 1934. That year he obtained the title of honorary professor. He died in 1937.

2 General characteristics of Natanson's scientific work

Results of Natanson's research were published in 144 papers and books; in Polish and foreign periodicals, mainly of Polish Academy of Sciences and Arts. (He was a member of the Academy from 1893). Almost whole his scientific work was devoted to theoretical physics. He published all his theoretical papers and books as the only author. His early experimental papers were the only exeption - he published them as a co-author. In spite of the studies with J.J. Thomson, in Cambridge and with Ludwig Boltzmann in Gratz he conssidered himself as a lonely authodidact. In his autobiography [2.1] he wrote:

Closing the general view on the introductory period of my life, I should say that the men who exerted the essential influence on my life were: in the childhood my Father and my older brother Edward; at the end of this period, as I mentioned above, Gosiewski and Witkowski. But in general I was and I remained an autodidact. Neither at school, nor at university, nor during the pilgrimage throughout the world l knew how to be and was able or was lucky enough to be a pupil. Already in this time I understood that it would be for me a large disadvantage in the future. Feeling regrets I saw (and now I see again) that I was autodidact and had all the shortages and faults of an autodidact. The deeper reason for this disadvantage lays in an intemal feature of my mind, which caused, that I was the true teacher for myself; I was a guide for myself, of course, a weak and unexperienced one. But I tried, throughout whole my life, to learn from the greatest masters, although they were dead for a long time. How much I leamed in this way from Newton, Lagrange, Kelvin, Clausius, J. W. Gibbs, G. G. Stokes, Lord Rayleigh, H. A. Lorentz, J. Clark Maxwell was always for me the very first, beloved example and leader.

Contact with work of great scientists always leaves traces and consequences in our mind and soul, which none lecture, seminar or foreign studies cannot exert.

The objects of intensive Natanson's research were the following branches of physics: Before the habilitation - the kinethic theory of gases, after the habilitation during the years 1893-1907 the thermodynamics of irreversible processes, its application to hydrodynamics of viscaous fluids, in the years 1907-1926 the theory of electrons, optics and the theory of radiation, in the thirties problems connected with quantum mechanics. The list of his papers in theoretical physics contains over 50 positions (most of them were published in Polish).

In the papers on thermodynamics of irreversible processes (published in 1896 and 1897) as well as in the paper on the statistics of quanta of radiation (1911) he went ahead of his epoch.

3 The only experimental research of Natanson

Young Władysław Natanson published two experimental papers with his older brother Edward. The first one was entitled *On dissociation of nitric dioxide*. The authors investigated a reversible chemical reaction

$$NO_4 \longleftrightarrow NO_2 + O2 \tag{1}$$

and veryfied the validity of the rule of Gouldberg's and Waage's for this reaction [3.1], [3.2].

In the other experimental paper Natanson with J. Boguski described an idea how to improve the construction of barometer [3.3].

All other Natanson's papers concerned theoretical physics were published by him alone.

4 Kinethic theory of gases

During the last years of the eighties Natanson considered the problems of the theory of gases. He was interested in three problems:

1) the kinetic explanation of Joule's effect (which was the subject of his doctor's dissertation) [4.1],

2) the problem of transition from the initial state of a gas to the equilibrium state, in which the distribution of the gas molecules' velocities is the Maxwell distribution [4.2],

3) the mechanism of dissociation of the electrolyte's molecules in a solution, namely whether the distribution is spontaneous or is a result of collisions with other molecules: Natanson stated that the second possibility is consistent with the experiment [4.3].

5 Thermodynamics of irreversible processes

During the years 1891-1899 Natanson worked in the domain of thermodynamics of irreversible processes. His papers on thermodynamics may be divided into three groups. The papers of the first group were written in connection with experimental work of Zygmunt Wróblewski and Karol Olszewski on determination of the conditions for liquefying hydrogen. Natanson determined the conditions of dynamical and statical liquefaction of hydrogen and explained why Wróblewski and Olszewski obtained the dynamical liquefaction only [5.1].

The second group Natanson's papers was devoted to development of traditional thermodynamics. He analysed the properties of four thermodynamical functions of state and formulated the condition for reversible or irreversible processes by taking entropy as an independent variable [5.2].

In the third, the most important group of papers on thermodynamics, Natanson investigated irreversible processes in which gases approach the equilibrium state. Natanson attempted to achieve understanding of those processes in two ways. The first way was based on investigations of the properties of the dissipation of energy function (introduced by lord Rayleigh) employing the methods of the kinetic theory of gases. Starting from the equation of the kinetic theory of gases he calculated the dissipation function of the fluid flow and discussed the ways of energy dissipation of viscous fluids. The second way consisted of attempts to generalise the contemporary termodynamics (which Natanson called thermostatics) into a theory including also irreversible processes; he proposed to give the new theory the name "thermokinetics".

The most important Natanson's paper on thermodynamics of irreversible processes, entitled *On the laws of irreversible processes* [5.3] was published in 1896. Natanson postulated validity of a certain variational principle in it. He called it *the thermokinetic principle.* It described thermodynamic processes in a similar way to Hamilton's principle in classical mechanics. (The forms of variational principles in traditional thermodynamics are presented in paper [5.4] by M. Kokowski).

Natanson's variational principle describing reversible and irreversible processes has the form:

$$\int_{t_0}^{t_1} \left(\delta T - \delta U + \sum_i P_i \delta q_i + \delta Q \right) dt = 0, \tag{2}$$

where t is time, T - kinetic energy which is a function of independent variables q and their derivatives with respect to time which he denotes by s_i, uniform in the second degree with respect to q_i and s_i, U is a potential energy depending on q_i, P is generalised external force in the direction of q_i, Q is the heat absorbed by the system, and is the sum of the compensated heat absorbed in reversible processes and the uncompensated heat of Clausius absorbed in irreversible processes

$$\delta Q = \delta_0 Q + \delta' Q. \tag{3}$$

Natanson deduced the properties of irreversible and reversible processes from this "thermokinethic principle" in a series of papers. He also defined the dissipation formfactor connected with irreversible processes and attempted to find its dependence on time.

6 Hydrodynamics of viscous fluids

The ideas of thermodynamics of irreversible processes found their application in hydrodynamics of viscous fluids. Natanson developed his theory of viscous fluids until the year 1907.

In his paper *On the laws of internal friction* [6.1] published in 1896 he formulated the assumptions of hydrodynamics of viscous fluids. He assumed that processes in viscous fluids are results of the interaction of two factors: inertia and coertia, which induces gradual adaptation to every deformation which is responsible for vanishing of tension. The vanishing of tension, or relaxation, can follow at appropriate velocity, measured by relaxation time. The concept of coertion, though not exactly precise, was Natanson's guide in this research. Natanson obtained the generalisation of the Navier - Stokes equation.

The results given in those papers evoked polemics with professor Zaremba, a mathematician from Jagellonian University.

Natanson's papers on thermodynamics on irreversible processes were not properly understood by contemporary physiciscts and chemists. Intensive development of the theory of thermodynamics of irreversible processes began thiry yers after publication (independently of Natanson) of two Lars Onsager's papers in 1931 [6.2], [6.3]. Onsager formulated thermodynamics of irreversible processes, in which thermodynamical fluxes (i.e. energy, mass, ...) are linear functions of stimulations, such as difference of temperatures, difference of tension, difference of concentration. But Onsager considered crossed processes and assumed the coefficients symmetry between stimula and fluxes (sometimes called the fourth law of thermodynamics) which permitted to consider connections between related processes.

7 Early papers in optics

Studies in the domain of hydrodynamics of fluids led Natanson to the problems of optics. Initially he was interested in optical properties of moving fluids. Later on, influenced by the success of Larmor and Lorcntz' theory of electrons, he continued his efforts of applying the theory of electrons to investigations of optical properties of gases and to radiation theory. The first of his papers dealing with these problems appeared in 1909, the last one in 1933.

In the paper *On fhe electromagnetic theory of dispersion and extinction in gaseous bodies* [7.1] Natanson considered, basing on the assumptions of the theory of electrons, the equation of motion of an electron influenced by the electric force of ingoing plane and monochromatic electromagnetic wave and by the polarization vector of the medium including also damping (the action of the magnetic field of the wave was neglected, what was permissible in the case considered by Natanson). Following Lorentz, Natanson assumed that damping of the movements of electrons is caused by mutual collisions of gas molecules and is proportional to the electrons velocities, but he neglected radiation damping, proportional to the second derivative of the velocity. He obtained the formulae for the coefficients of refraction and extinction, known later as "Drude-Natanson rule" which stated that the product $a\Delta$ (where a is the number of optically active electrons in the molecule and $\Delta = 3\pi cm/e^2N$, where c is the velocity of light, m is the electron mass, e is the electron charge and N is the number of optically active electrons in 1 cm^3 of gas in normal conditions), is a universal constant, i.e. has the same value for all kinds of molecules. Further the author verified his formula versus the then available experimental data. He obtained a rather good agreement for hydrogen, air, carbon oxide, ammonia, but he noted a disagreement for such gases as carbon dioxide or noble gases.

In the paper *On the elliptic polarization of light transmitted through an arbitrary gaseous medium and parallel to the lines of an external magnetic field* [7.2] Natanson investigated the phenomena of transmission of the circularly polarised plane electromagnetic wave through gaseous medium located in a constant magnetic field parallel to the direction of the wave propagation. Applying the equations of motion for an electron moving under the influence of the electromagnetic field of the ingoing electromagnetic wave and of the extcrna1 magnetic field, the author found that circularly left and right polarised waves have not only different refraction coefficients, but also different amplitudes. The linearly polarised wave falling on the considered medium decomposes into a superposition of left and right circularly polarized waves, having different velocities and different extinctions. When leaving the medium, these waves combine into elliptically polarized wave, Natanson obtained the same effect in the next paper *On elliptic polarization of light transmitted through an absorbing naturally active medium* [7.3] investigating the transmission of the plane electromagnetic wave in an optically active medium (rotating the polarization plane). He solved equations of motion of the electron in this case, assuming that besides the electromagnetic field a torsional force is acting, which is proportional to the curl of electric field and as the result obtained that the linearly polarized wave falling on such medium leaves it as a circularly polarized wave. Natanson was the first to predict such influence of extinction on the phenomena of transmission of light through matter; since the time of Fresnel nobody suggested such a possibility. The results of this paper were discussed in a supplementary publication where 'Natanson noticed that Babinet's rule, according to which the less absorbed light waves are faster, is not always true. Then he compared his results with Cotton's experiments and found good agreement.

Natanson returned to the comparison of this theory with experiment by investigating the agreement of Drude - Natanson's rule with the results of experiments performed in 1908 and 1909. The discussion of this research was published in the paper *Note on the theory of extinction in gaseous bodies* [7.4]. Natanson obtained similar results there as in the paper on electromagnetic theory of dispersion and extinction in gaseous bodies. Let us remark that in [7.4] he also quoted the results of experiments of Stanisław Loria, who was then assistant in experimental physics in Cracow.

In the paper *On the theory of extinction in gaseous bodies* [7.5] Natanson nodified his theory of transmission of electromagnetic wave through a gaseous medium. He neglected the damping term, considered previously in equations of motion, which was proportional to the velocity and instead he included the influence of radiation damping, introducing the term proportional to the second derivative of velocity into equations of motion of the electron. The relations between the refraction coefficient and coefficient of extinction obtained in this paper are formally identical with those of [7.1] but the formulae contained here have the coefficient of radiation damping, which depends only on universal constants. From these relations Natanson obtained, in a simple manner, Rayleigh's formula for the dependence of the radiation energy loss in the gas on the wave length of light. The last part of the paper was devoted to the comparison of Rayleigh's formula with measurements performed by many authors. It turned out, that Rayleigh's formula agreed with experiment, whereas disagreement was obtained when extinction of light wave was calculated from the Lorentz' theory, according to which the damping of electron movements was caused by collisions of molecules; his comparison yields strong evidence against Lorentz" theory.

In the last publication of this series, performed before 1910, entitled *On the theory of double refraction induced by an electric and magnetic field* [7.6] Natanson investigated Kerr and Cotton-Mouton effects by the methods of the theory of electrons.

8 Paper and monographs on quantum statistics

In 1911 Natanson turned his scientific interest towards quantum theory. The papers *On the statistical theory of radiation* [8.1] published in that year belong to the most important Natanson's publications. They contain the first formulation of quantum statistics, which, 13 years later was rediscovered independently by an Indian physicist Bose [8.2], developed by Einstein and is today called "Bose - Einstein statistics". In the paper quoted above, Natanson investigated the derivation of energy distribution of black body radiation, described by Planck's law using the principles of statistical mechanics. This problem was already treated by Planck himself and also by Einstein, Larmor, Wilson, Debye and Lorentz. Natanson was the first to realize that photons (which he called "material units") are indistinguishable objects, which can exist only in discrete energy states. The energy distribution of the ensemble of indistinguishable particles differs essentially from that of distinguishable particles (which are considered in the classical kinetic energy of gases). Natanson calculated the most probable energy distribution of photons and obtained the law of energy distribution in an implicit form. Natanson's formula yields for the limiting case of all energy states occupied by many particles the Maxwell-Boltzmann distribution, and for the opposite case of energy states occupied by few particles only, the Planck distribution. Let us remark that Planck, having the same initial distribution as Natanson, did not attract the reader's attention to the fact that the particles considered are indistinguishable and did not try to determine the most probable state. A historian Armin Hermann expressed the opinion that Natanson found himself, besides Max Planck, Albert Einstein and Paul Ehrenfest among the first scientists who formulated the principles of quantum statistics and that his merits had not still been properly recognized by historians of physics.

The fact of underestimation of Natanson's discussed papers can be partly explained by the fact that they were written in the form, which was difficult to understand, and also by the fact that Natanson did not connect the energy of the photon with its frequency. This has been done later by Bose, who divided the momentum space of the photons into the cells of the volumes h^3, took into account two possibilities of linear polarization of the photon and obtained the Planck's law in the form, in which universal constants occur.

Natanson applied the method developed in the paper on statistical radiation theory to the calculation of the energy of the solid body. In the paper *On energy content in material bodies* [8.3] he considered, according to Einstein, the solid body as a system of oscillators vibrating with various frequencies. By means of his method Natanson obtained the formula for the energy of solid body identical with the law obtained by Einstein in his theory of the specific heat of solids. Having finished these papers Natanson wrote (in Polish) a monograph entitled *Principles of radiation theory* [8.4]], in which he presented the outline of this theory starting from Kirchoff's research initiated in 19th century, then presented the Planck's work, as well as his own results. An analysis of Natanson's paper [8.1] was made by J. Spałek [8.5].

9 Later papers in optics

Having published those papers Natanson returned to the problem of transmission of light through gases. In this domain he was busy from 1916 to 1926, besides his work on the theory of electrons, despite the fact that there had already existed the "old" quantum theory developed in the first decades of the 20th century. Natanson accepted a critical standpoint towards it. He expressed his opinion in the letter to professor A. Rubinowicz (citied by R.S. Ingarden) [9.1]:

Did Laplace, Fourier and Ampère construct their theories like that [like old quantum theory]? I confess, in spite of great respect to Planck, Einstein, Ehrenfest, Sommerfeld etc, I would be very happy if one day the quantum theory disappeared totally from physics. Perhaps these gentlemen would also be content.

Results of Natanson's research on light transmission through gases were published in four papers. In the first of them *On the scattering of light in gaseous medium* [8.2] he developed the idea of Lord Rayleigh, who explained the blue colour of the sky as due to the scattering of solar light by small particles of dust floating in the atmosphere. Natanson investigated scattering of the plane monochromatic and linearly polarized electromagnetic wave by the gas composed of molecules, considered as damped linear oscilators vibrating independently with the same frequency. They are supposed to have electric charge and therefore they emit spherical electromagnetic waves under the influence of the ingoing wave. The energy needed to produce these spherical waves is scattered in all directions and therefore the ingoing wave is weakened. Natanson calculated the energy loss of the wave passing through a layer of gas and obtained a formula, which for weakly damped oscillators can be reduced to the Rayleigh's law, according to which the energy loss of ingoing wave is inversely proportional to the fourth power of the wavelenght. It has to be stressed, however, that despite obtaining the Rayleigh's formula, the scattering mechanism considered by Natanson did not explain definitely the phenomenon of the blue colour of the sky, since, as it was explained by Smoluchowski and Einstein, it is due to the scattering of light on density fluctuations of the gas particles. (N. b. this fact was known to Natanson, who in his paper citied Einstein's publication, which was a direct supplement to Smoluchowsk's paper on density fluctuations written in 1908).

The theory presented in the paper quoted above was further developed by Natanson in his next papers. In the paper *On the molecular theory of the reflection of light* [9.3] he considered the transmission of plane monochromatic electromagnetic wave through the gas (composed of oscillators described previously), limited from one side by a plane. Applying his method he derived the expression for electric and magnetic vectors of the transmitted and reflected waves as well as expressions for these vectors in the case of absolute reflection of light when the wave propagates from the gas interior. These expressions agree with those which could be obtained from classical optics if the continuous medium transmitting the electromagnetic wave is considered and if boundary conditions are imposed.

In the paper *On the propagation of energy conveyed by radiation through material media* [8.4] Natanson calculated the Poynting vector for plane monochromatic light waves passing through a layer of gas considered as an ensemble of linear oscillators and evaluated the dissipation function for this wave. Then he considered the packet of electromagnetic waves passing through the gas and calculated its group velocity.

In the next paper of this series entitled *Secondary radiation from simple rotators* [9.5] Natanson considered the model of a gas consisting of rigid plane rotators (or rigid electric dipoles), penetrated by a plane, circularly polarized electromagnetic wave. Applying the method of his earlier publications Natanson calculated electric and magnetic vectors and the Poynting vector of a wave scattered by a rotator. It turned out that the process of transmitting of electromagnetic wave by an ensemble of rotators is a complicated effect in which diverse beams of scattered radiation are produced. But when the transmission of light becomes stationary, these beams mutually extinguish and in such a medium there follows a quick extinction of the falling wave.

In the paper *On the molecular theory of refraction, reflection and extinction* [9.6] Natanson generalized the method applied in his paper on the molecular theory of the reflection of light by taking into account the effect of the damping of radiation and calculated the coefficients of refraction, reflection and extinction of light by a stratum filled with molecules, treated as non-interacting and randomly damped harmonic oscillators. His scientific interests were directed more and more to the history of science. The number of Natanson's papers on theoretical physics became smaller than in previous periods. In the twenties Natanson continued his study on scattering and absorption of radiation by gaseous media and published two papers on this subject. The paper *On the theory of molecular scattering and extinction of light* [9.7] was an improved version of two previous Natanson's papers [7.5] and [7.3].

In the last paper of this series, entitled *Vibration around the steady motion* [9.8] Natanson presented shortly the history of the attempts to explain the phenomenon of dispersion of light, and described a certain mechanical model illustrating this phenomenon. In this model light wave provokes the electron oscillations not around the position of equilibrium, but disturbs the steady motion performed by electron before the light wave fell on the molecule. The author showed that for oscillating steady motion the hypothesis of vibration around steady motion leads to the same result as the one obtained by considering vibration around equilibrium state. The above mentioned papers ended in 1926 the long series of Natanson's papers based on the theory of electrons. Natanson enthusiastically accepted quantum mechanics formulated in the years 1925-1929. He was especially interested in the wave aspect of matter expressed in this theory.

He published five papers containing his original way of research by investigating the connection between, generalised by himself, Fermat's principle of geometrical optics with classical mechanics and quantum theory. He presented the results of his research in a textbook *First principles of undulatory Mechanics* [9.9] and in the paper *Fermat's Principle* [9.10].

10 Papers and books on history and methodology of physics

Natanson's interests in the essence and limits of scientific recognision and his vivid interest in history of science were reflected in a series of articles and lectures written in Polish. The majority of them is collected in four books entitled *Lectures and sketches* [10.1], *The Face of Nature* [10.2], *The Order of Nature* [10.3] and *Horizon of Science* [10.4].

The booklet *Lectures and Sketches* contains a series of five lectures delivered in the years 1902-1907. Natanson discussed there such topics as inertia and coertia, the relation of thermodynamics to the kinetic theory of matter, the theory of magnetism and the theory of electrons. The same problems, but formulated more generally and in more modern way, as well as the texts of his speeches delivered on the occasion of various festivities and conferences are contained in the second collection of Natanson's essays and articles *The face of Nature.* Here the article about the blue colour of sky deserves particular attention. The book *Horizon of Science* contains many essays on the history of science; the author discussed there Greek philosophy, the work of Lucretius, life and work of Faraday, Maxwell and Cavendish. In last years of his life Natanson was interested in the development of scientific thought in the Islamic world in Middle Ages. He published a book *Intellectual trends in the early Islam* [10.5], which appeared few weeks before his death. The selection of his essays was published in 1977 as *Reminiscences and sketches* [10.6] prepared and commented by A. Piekara.

According to Natanson, science is born from deepest needs of the human nature, aiming to understand the surrounding world. But, according to J. Dąmbska [10.7], Natanson assumed the attitude of cautious criticism (or Kant's epistemological criticism) towards the process of cognition. According to Natanson, cognitive process leads to construction of scientific theory and this construction depends on the scientist performing the investigation. When the region of the research grows, the ways of formulation and explaining of the recognized facts change, but one cannot penetrate through the world of phenomena to the reality lying behind it. The process of scientific research cannot be restricted by the assumption of any metaphysical system (such as materialism, idealism or dualism). But otherwise one should not trust the actual physical theorems, but one must bear in mind their hypothetical and approximate character and the fact that they can soon lose their validity.

11 Pedagogical activity

Natanson obtained veniam legendi in 1892, and he started to lecture theoretical physics in the academic year 1892/93 and continued until 1933/34, except for the year 1914/1915, which because of military action during the World War I he spent with his family in Berlin (During that time he maintained vivid scientific contacts with Einstein).

Natanson lectured theoretical physics in 4-6 years' cycles embracing mechanics, hydrodynamics, kinethic theory of gases, electrodynamics, optics, theory of electron and radioactivity. He also lectured theory of relativity and in the year 1930/31 he gave the first lecture of quantum mechanics in Cracow.

Leopold Infeld, who listened to Natanson's lectures in 1913-1920 wrote in his memoir: *I have listened to many beautiful lectures in my life, but never as technically perfect as the lectures of professor Natanson.*

Natanson also wrote textbooks. The first of them, *Introduction to Theoretical Physics* [1.8], appeared in 1890. It contained a systematic exposition of classical physics. In 1891 and 1895 he wrote two textbooks for elementary schools [11.1]. In 1901 there appeared *Theoretical mechanics* [11.2], a textbook as the first volume of a vademecum series for auto-didacts. In 1913 he gathered the laws of radiation in a textbook *Foundations of the theory of radiation* [8.1]. For some years he collaborated with professor K. Zakrzewski preparing a textbook of physics for gymnasia [11.3], which appeared in the years 1922-1925. The last Natanson's textbook of physics entitled *The first Foundations of Undulatory Mechanics* [9.9] appeared in 1930.

During the years 1922-1928 Natanson put a lot of effort into editing Smoluchowski's collected papers (Smoluchowski died in1917). Natanson edited *Marian Smoluchowski's Collected Papers in Three Volumes* [11.4] in those years. The first volume appeared with collaboration of Jan Stock. After Stock's death in 1925 the second and third volumes were prepared by Natanson alone.

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