Comment on
“TIME OPERATOR”: THE CHALLENGE PERSISTS

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Abstract

The criticism formulated by Mielnik and Torres-Vega in their paper, does not apply to my derivation of the time of arrival in Quantum Mechanics but to our present understanding of the quantum measurement theory.

In paper [1] I have expressed the following opinion:

Although, there is no room to modify the mathematically unique definition of the probability $Q_{I \times D}(\psi)$ within the standard mathematical framework of Quantum Mechanics (spectral measures, self-adjoint operators etc.), the problem is still open from the physical point of view. Indeed, the probabilistic interpretation of the quantum mechanics is based on “short” measurements of the type “What is a probability that the particle will be found within the three-dimensional volume $V$ precisely at a given time instant $t$?” How to relate them with “long” measurements, corresponding to time intervals rather than to time instants $t$ is by no means obvious.

Bogdan Mielnik and Gabino Torres-Vega propose a deep analysis of conceptual problems which arise when one starts to inter-
pret “long” measurements in terms of the conventional framework of Quantum Mechanics. To strengthen the dramatic effects, they decided to introduce a “scapegoat” as the main *dramatis persona*. I am deeply touched by the fact, that this role has been assigned to my humble self. The prize I am proposed to get for this job is indeed high: I am allowed to consider myself, together with P. A. M. Dirac and J. von Neumann, a member of the “gang of three” – those who petrified the present “hard orthodoxy” in Quantum Mechanics, oppressing free spirits, like, e. g., Bogdan Mielnik.

The idea to become a member of an exclusive society, together with Dirac and von Neumann is, of course, delightful. But the entire deal is unfair and I do not like it at all. During the last 30 years, since I have published my result for the first time in 1974, we have had many discussions with Bogdan Mielnik. He knows very well, that I never considered my result as a physically satisfactory solution of the problem and the *passus* from my paper which opens this letter proves it undoubtedly. My construction of “arrival time” is indeed *mathematically unique and final* within the conceptual framework of the standard interpretation of Quantum Mechanics. But I always considered it as an argument for further analysis of the conceptual framework of the theory. The physical arguments given by Mielnik and Torres-Vega against this construction are just arguments against Quantum Mechanics in its present shape. Exactly the same arguments may be formulated against, e. g., Newton-Wigner position operator in relativistic quantum mechanics, which is again unique in the same mathematical framework (as proved by J. Kijowski and G. Rudolph, see [2]). Here, the list of physical problems is equally long. For example: if a quantum state is localized within a tiny space region in one reference frame, it is generically spread over the entire space in another reference frame. What seems to be a friendly “instantaneous measurement” in one reference frame, becomes a nasty “waiting detector” in another one.

Insisting on the self-adjointness of operators is not a political repression used by those nasty (orthodox!) mathematicians in order to restrict somebody’s freedom of expression. A self-adjoint operator implements the very notion of a simplest *quantum observable*: an increasing family of projectors $E(\lambda)$ labelled by possible values $\lambda$ of the measurement. Unfortunately, at the moment there is no measure-
ment theory, which could replace this (naive and very unsatisfactory!) picture. I wish Bogdan Mielnik to find one.

If we want to improve physical predictions of quantum mechanics (i.e. that the atom in 50-th Rydberg state will remain *for ever* in this state) by taking into account radiation phenomena, we must replace quantum mechanics by the quantum field theory. But here, the situation is even worse. What is good, is that quantum observables are local and they fulfill local field equations. But the very nature of a quantum state is *highly non-local* not only with respect to space (this problem is common with quantum mechanics), but also with respect to time: in general, no observable may be assigned to a *sharp* instant of time, but must be smeared over a time interval. Thus, we are unable to give any rigorous meaning even to “instantaneous measurements” (cf. [3] and [4]).

The analysis of the problem, done by Mielnik and Torres-Vega is, in my opinion, very interesting. On the other hand, I do not accept the role of a scapegoat. Some time ago V. Delgado, J. G. Muga (see [5]) attacked me for being not sufficiently orthodox: *Our result turns out to be similar to those previously obtained by Kijowski. However, the approach by Kijowski was based on the definition of a non-conventional wave function ... whose relation to the conventional wave function is unclear.*

A similar critique was expressed by N. Grot, C. Rovelli and R. S. Tate (see [6]).

Now, Mielnik and Torres-Vega criticize my “hard orthodoxy”. I consider the above ideological attacks from contradictory directions as fully unjustified: in paper [7] I have just proved a mathematical theorem and nobody has questioned this result.

At this point one should notice that replacing spectral measures $E(\lambda)$ by positive operator valued measures (POV measures, see [8]) does not change essential features of the “orthodox” measurement theory but allows for a slightly more sophisticated experimental setting, where even the physical interaction which we use for the measurement depends upon the range of $\lambda$ we are investigating. Within this framework R. Brunetti and K. Fredenhagen were able to prove the time-energy uncertainty principle (and even more!) without any specific realization of the “arrival time” observable. But again, this is a nice mathematical result, which does not answer any of physical
questions discussed by Mielnik and Torres-Vega.

Finally, let me point out just one misunderstanding: the observable \( \Theta \), so much ridiculed by the authors (see their formula (3) and the text below), gives the correct time value on 100% classical observables, provided we measure it for the “right-movers” and the “left-movers” separately, which we always do in the experiment.

References


